



Semester One Examination, 2021

Question/Answer booklet

MATHEMATICS METHODS UNIT 1



Section Two: Calculator-assumed

WA student number: In figures

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In words

Your name

Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

Number of additional
answer booklets used
(if applicable):

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Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed

65% (98 Marks)

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9

(6 marks)

A survey of all apartments for sale in a particular suburb showed that 65% had a lift and 78% had a swimming pool. Of those with a lift, 80% had a swimming pool.

Determine the probability that a randomly selected apartment from those in the survey had

- (a) a lift and a swimming pool. (2 marks)

Solution
$P(L \cap S) = 0.65 \times 0.80$ $= 0.52$
Specific behaviours
<ul style="list-style-type: none"> ✓ indicates correct method ✓ calculates probability

- (b) a lift or a swimming pool. (2 marks)

Solution
$P(L \cup S) = 0.65 + 0.78 - 0.52$ $= 0.91$
Specific behaviours
<ul style="list-style-type: none"> ✓ indicates correct method ✓ calculates probability

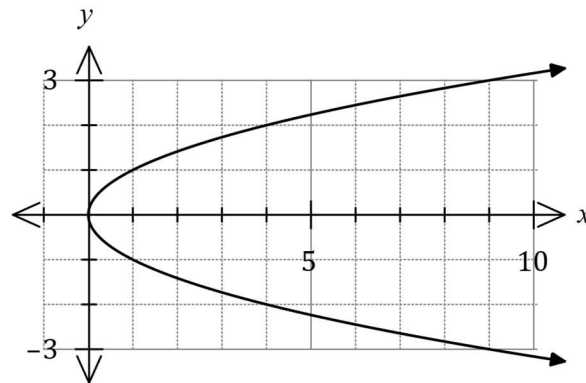
- (c) no lift or no swimming pool. (2 marks)

Solution
$P(\bar{L} \cup \bar{S}) = 1 - P(L \cap S)$ $= 1 - 0.52$ $= 0.48$
Specific behaviours
<ul style="list-style-type: none"> ✓ indicates correct method ✓ calculates probability

Question 10

(6 marks)

(a) The parabolic graph of a relation is shown below.



(i) State the equation of its axis of symmetry. (1 mark)

Solution
$y = 0$
Specific behaviours
✓ correct equation

(ii) State the equation of the relationship between x and y . (1 mark)

Solution
$x = y^2$
Specific behaviours
✓ correct equation

(b) Points A and B have coordinates $(7, 8)$ and $(-3, 2)$ respectively. Determine the equation of the circle that has diameter AB . (4 marks)

Solution
Midpoint: $\left(\frac{7-3}{2}, \frac{8+2}{2}\right) = (2, 5)$
Radius: $r^2 = (7-2)^2 + (8-5)^2$ $= 34$
Equation: $(x-2)^2 + (y-5)^2 = 34$
Specific behaviours
<ul style="list-style-type: none"> ✓ identifies midpoint as centre ✓ calculates coordinates of centre ✓ calculates square of radius ✓ writes equation of circle

Question 11

(8 marks)

- (a) Triangle ABC is such that $b = 25$ cm, $c = 33$ cm and $\angle A = 142^\circ$. Determine, with justification, the length of side a . (2 marks)

Solution
$a^2 = 25^2 + 33^2 - 2(25)(33) \cos 142^\circ$ $a = 54.9 \text{ cm}$
Specific behaviours
<ul style="list-style-type: none"> ✓ clearly shows use of cosine rule ✓ correct length

- (b) Triangle PQR is such that $p = 41.2$ cm, $q = 55.3$ cm and $\angle P = 33^\circ$. Determine all possible areas of this triangle. (6 marks)

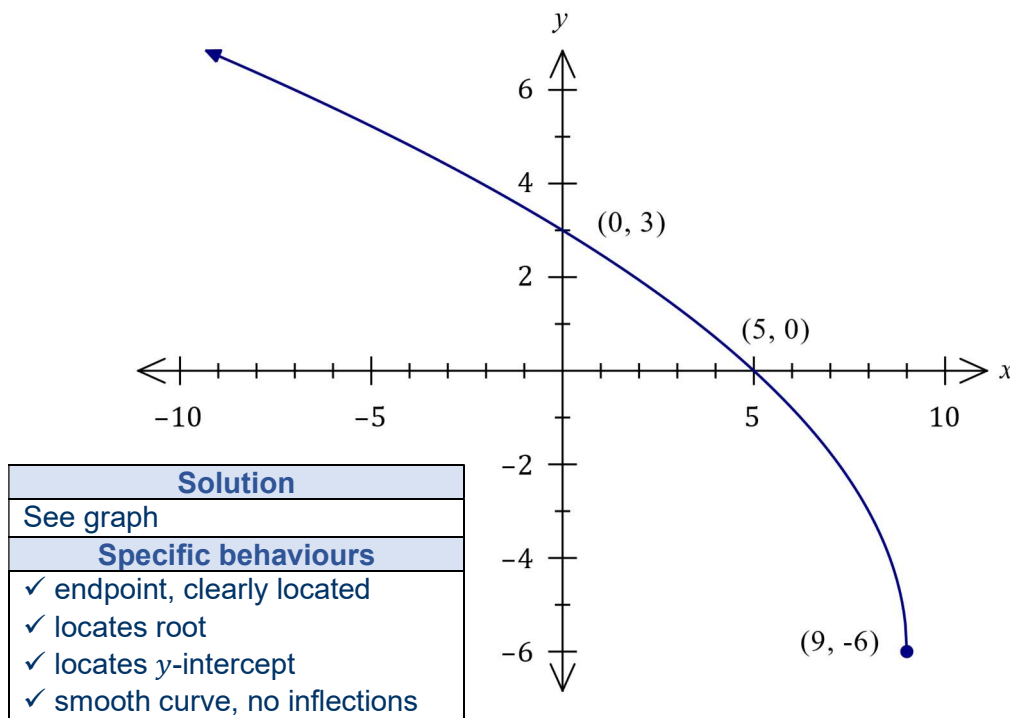
Solution
$\frac{\sin Q}{55.3} = \frac{\sin 38^\circ}{41.2}$
<p>First solution:</p> $\angle Q_1 = 47^\circ$ $\angle R_1 = 180^\circ - 33^\circ - 47^\circ = 100^\circ$ $A_1 = \frac{1}{2}(55.3)(41.2) \sin 100^\circ$ $A_1 = 1122 \text{ cm}^2$
<p>Second solution:</p> $\angle Q_2 = 180^\circ - 47^\circ = 133^\circ$ $\angle R_2 = 180^\circ - 33^\circ - 133^\circ = 14^\circ$ $A_2 = \frac{1}{2}(55.3)(41.2) \sin 14^\circ$ $A_2 = 275 \text{ cm}^2$ <p style="text-align: center;">Areas are 275 cm^2 and 1122 cm^2.</p>
Specific behaviours
<ul style="list-style-type: none"> ✓ shows use of sine rule ✓ calculates one value for $\angle Q$ ✓ derives $\angle R$ and shows use of area formula ✓ calculates one correct area ✓ calculates second set of values for $\angle Q$ and $\angle R$ ✓ calculates second area

Question 12

(8 marks)

Let $f(x) = 3\sqrt{9-x} - 6$.(a) Sketch the graph of $y = f(x)$ on the axes below.

(4 marks)

(b) Describe the transformation(s) required to obtain the graphs of the following functions from the graph of $y = f(x)$:

(i) $y = \sqrt{9-x} - 2$.

(2 marks)

Solution
$y = \frac{1}{3}f(x)$. Vertical dilation of scale factor $\frac{1}{3}$.
Specific behaviours
✓ both bolded words in description
✓ correct scale factor

(ii) $y = 3\sqrt{1-x} - 6$.

(2 marks)

Solution
$y = f(x - 8)$. Horizontal translation of 8 units to the left.
Specific behaviours
✓ states a translation
✓ correct distance and direction

Question 13

(8 marks)

A study of the achievements of 360 students enrolled in a university course yielded the following information:

- 50% of all students achieved a distinction
- 60% of those who did not achieve a distinction studied part-time
- 45% of those who studied full-time did not achieve a distinction

(a) Use the above information to complete the following table. (4 marks)

	Full-time	Part-time	Totals
Distinction	88	92	180
No distinction	72	108	180
Totals	160	200	360

Solution		
$360 \times 0.5 = 180,$	$180 \times 0.6 = 108,$	$180 - 108 = 72$
$\frac{72}{x} = 0.45 \Rightarrow x = 160,$	$160 - 72 = 88,$	$180 - 88 = 92$
Specific behaviours		
✓ splits total to D/ND; ✓ splits no distinction to FT/PT		
✓ calculates PT total; ✓ completes table		

(b) Determine the probability that a randomly chosen student from the study

(i) achieved a distinction and studied full-time. (1 mark)

Solution	
$P = \frac{88}{360} = \frac{11}{45} = 0.2\bar{4}$	
Specific behaviours	
✓ correct probability	

(ii) achieved a distinction or studied full-time. (1 mark)

Solution	
$P = \frac{360 - 108}{360} = \frac{252}{360} = \frac{7}{10}$	
Specific behaviours	
✓ correct probability	

(c) Explain whether this study provides any evidence that achieving a distinction is independent of study mode. (2 marks)

Solution	
$P(D) = 0.5$	
$P(D FT) = 1 - 0.45 = 0.55$	
Hence events are not independent as $P(D) \neq P(D FT)$.	
Specific behaviours	
✓ states $P(D)$ and a conditional probability	
✓ states not independent with reason	

Question 14

(8 marks)

A souvenir shop sells T-shirts in two colours and three sizes. Sales records for the past year are shown below.

	Small	Medium	Large
White	180	230	210
Green	170	450	260

Assume that the shop holds a large stock and that sales continue in similar proportions. Where relevant, round your answers in this question to three decimal places.

(a) A customer randomly selects a T-shirt for purchase. Determine

- (i) the least likely size and colour of this T-shirt and the probability that this T-shirt is selected. (2 marks)

Solution
Least likely: Small, green.
$P = \frac{170}{1500} = \frac{17}{150} \approx 0.113$
Specific behaviours
<ul style="list-style-type: none"> ✓ type of T-shirt ✓ calculates probability

- (ii) the probability that the T-shirt selected is not a large. (2 marks)

Solution
$P = \frac{350 + 680}{1500} = \frac{1030}{1500} = \frac{103}{150} \approx 0.687$
Specific behaviours
<ul style="list-style-type: none"> ✓ counts required sizes ✓ calculates probability

(b) A customer randomly selects two T-shirts for purchase. Determine the probability that the T-shirts are

- (i) both small. (2 marks)

Solution
$P(SS) = \left(\frac{350}{1500}\right)^2 = \frac{49}{900} \approx 0.054$
Specific behaviours
<ul style="list-style-type: none"> ✓ probability of one small ✓ calculates probability

- (ii) of different colours. (2 marks)

Solution
$P(WG) = \frac{620}{1500} \times \frac{880}{1500} = \frac{1364}{5625} (\approx 0.2425)$
$\therefore P(WG \cup GW) = 2 \times \frac{1364}{5625} = \frac{2728}{5625} \approx 0.485$
Specific behaviours
<ul style="list-style-type: none"> ✓ probability of one then the other ✓ calculates probability

Question 15

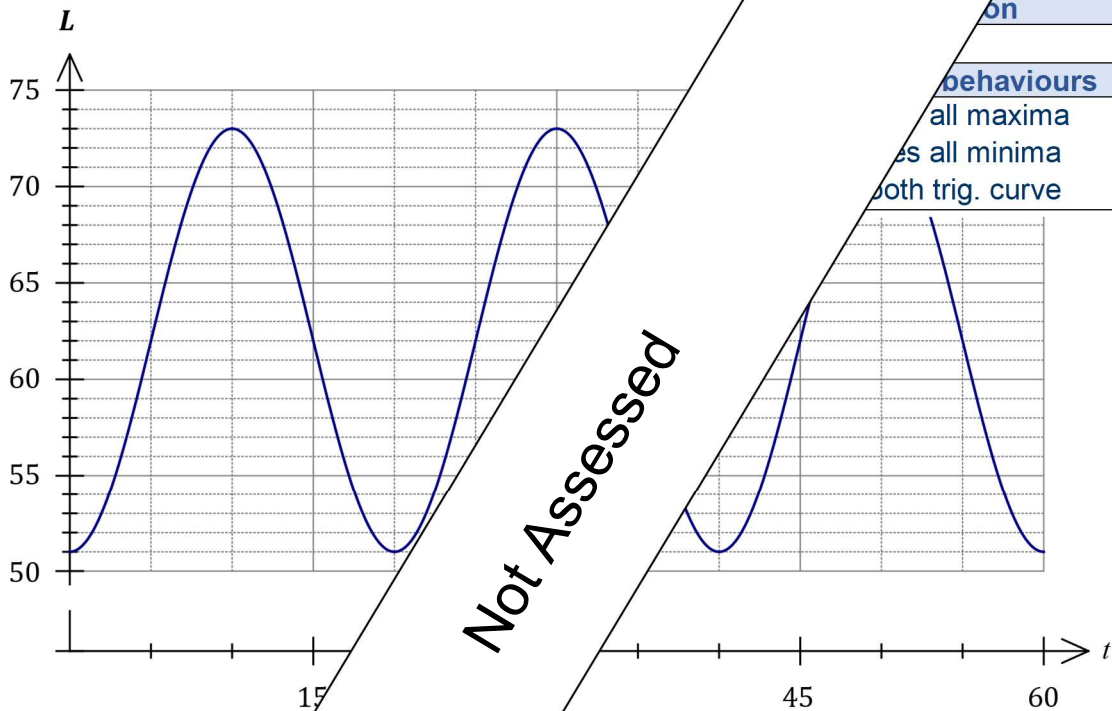
The loudness L of sound, in decibels, emitted by a machine t minutes after it is switched on can be modelled by

$$L = 62 - 11 \cos\left(\frac{\pi t}{10}\right)$$

- (a) Determine the initial loudness emitted by the machine. (1 mark)

Solution
$L(0) = 51$ dB
Specific behaviours
✓ correct value

- (b) Draw the graph of L against t on the axes below for the first 60 minutes. (3 marks)



on
behaviours
all maxima
all minima
both trig. curve

- (c) State the maximum loudness reached by the machine and the time this maximum was first reached. (2 marks)

Solution
73 dB when $t = 10$ s.
Specific behaviours
correct maximum
correct time

- (d) A health regulator can deem a machine unserviceable if the loudness it emits exceeds 73 dB for more than 15 minutes in any hour that it is running. Determine, with justification, how many hours this machine could be deemed unserviceable. (3 marks)

Solution
needs value for $73 < L < 75$ during first cycle.
$3(12.41 - 7.59) = 14.5$ minutes per hour - and so machine could not be deemed unserviceable.
Specific behaviours
✓ identifies interval endpoints
✓ calculates minutes per hour
✓ uses calculations to draw conclusion

Question 16

(8 marks)

- (a) Let $f(x) = x^2 + bx + c$, where b and c are constants. The graph of $y = f(x)$ has an axis of symmetry with equation $x = 4$ and an axis intercept at $(0, 6)$.

- (i) State the value of the constant c . (1 mark)

Solution
c is the y -intercept: $c = 6$.
Specific behaviours
✓ correct value

- (ii) Determine the value of the constant b . (2 marks)

Solution
Axis of symmetry has equation $x = -\frac{b}{2a}$: $4 = -\frac{b}{2} \Rightarrow b = -8$
Specific behaviours
✓ indicates appropriate method ✓ calculates value

- (b) Let $g(x) = -(x + 3)^2 + 5$. Determine

- (i) the coordinates of the turning point of the graph of $y = g(x)$. (1 mark)

Solution
Turning point is at $(-3, 5)$.
Specific behaviours
✓ correct coordinates

- (ii) the domain and range of $g(x)$. (2 marks)

Solution
Domain: $x \in \mathbb{R}$, and range: $y \leq 5$.
Specific behaviours
✓ states domain ✓ states range

- (iii) the coordinates of the turning point of the graph of $y = g(x + 2) - 3$. (2 marks)

Solution
Graph has been translated 2 units left and 3 units downwards and so new turning point at $(-5, 2)$.
Specific behaviours
✓ indicates correct use of one translation ✓ correct coordinates

Question 17

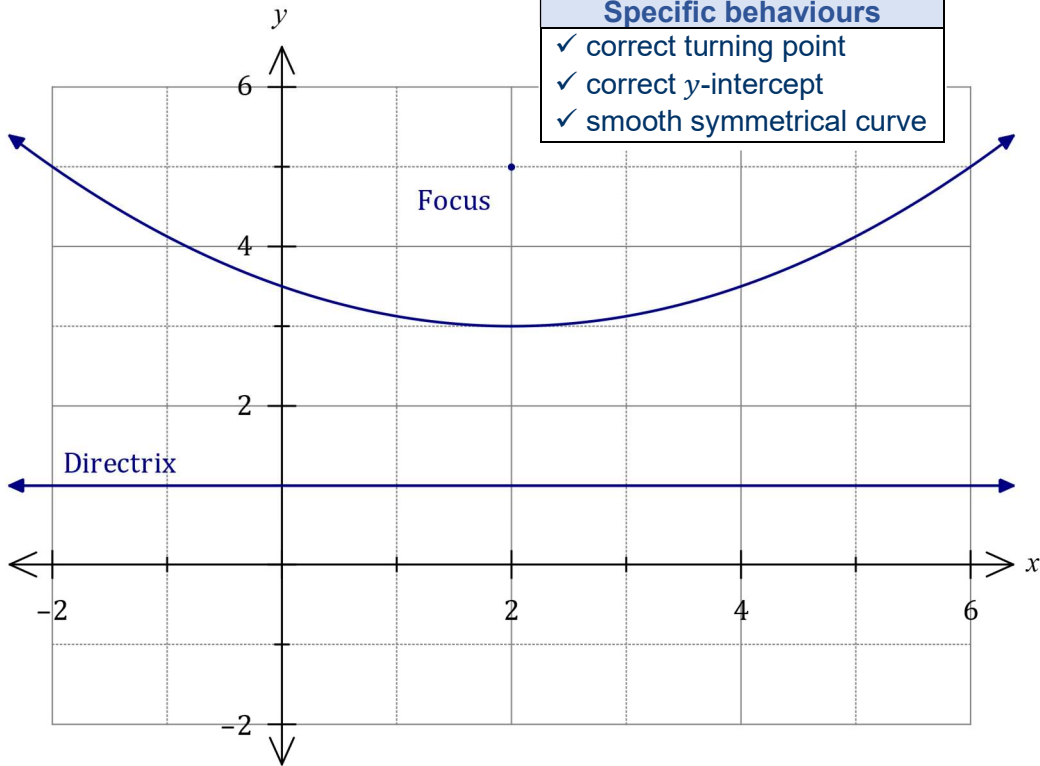
(7 marks)

The equation of a parabola is $y = \frac{1}{8}(x^2 - 4x + 28)$.

(a) Sketch the parabola on the axes below.

Solution
See graph
Specific behaviours
✓ correct turning point
✓ correct y-intercept
✓ smooth symmetrical curve

(3 marks)



All parabolas have a focal point and a directrix. For a parabola with equation $y = a(x - p)^2 + q$, the focal point is at $(p, q + \frac{1}{4a})$ and the equation of the directrix is $y = q - \frac{1}{4a}$, where a, p and q are constants.

(b) Determine the focal point and directrix for this parabola and add them, with labels, to your sketch above. (4 marks)

Solution
From graph, turning point at (2, 3). Hence
$a = \frac{1}{8}, \quad p = 2, \quad q = 3$
Focal point: (2, 5) and directrix: $y = 1$.
Specific behaviours
✓ indicates turning point
✓ indicates values of all constants
✓ plots focus
✓ draws directrix

Question 18**(8 marks)**

The events A and B are such that $P(A) = 0.4$ and $P(B) = 0.5$.

Determine $P(A \cup B)$ in each of the following cases:

(a) A and B are mutually exclusive.

(1 mark)

Solution
$P(A \cup B) = 0.4 + 0.5 = 0.9$
Specific behaviours
✓ correct probability

(b) $P(A \cup \bar{B}) = 0.75$.

(2 marks)

Solution
$P(B \cap \bar{A}) = 1 - 0.75 = 0.25$
$P(A \cup B) = 0.4 + 0.25 = 0.65 = \frac{13}{20}$
Specific behaviours
✓ calculates $P(B \cap \bar{A})$
✓ calculates probability

(c) A and B are independent.

(2 marks)

Solution
$P(A \cap B) = 0.4 \times 0.5 = 0.2$
$P(A \cup B) = 0.4 + 0.5 - 0.2 = 0.7$
Specific behaviours
✓ calculates $P(A \cap B)$
✓ calculates probability

(d) $P(B|\bar{A}) = \frac{7}{12}$.

(3 marks)

Solution
$P(B \bar{A}) = \frac{P(B \cap \bar{A})}{P(\bar{A})}$
$P(B \cap \bar{A}) = (1 - 0.4) \times \frac{7}{12} = 0.35$
$P(A \cup B) = 0.4 + 0.35 = 0.75$
Specific behaviours
✓ derives conditional probability relationship
✓ calculates $P(B \cap \bar{A})$
✓ calculates probability

Question 19

(8 marks)

A chess club has 12 members, of which 5 are beginners, 3 are intermediate and the rest are advanced. The club has to select a group of 4 members at random to assist with a regional tournament.

(a) Determine the number of different groups that can be selected.

(2 marks)

Solution
$\binom{12}{4} = 495$
Specific behaviours
<ul style="list-style-type: none"> ✓ correctly uses any combination notation ✓ correct number

(b) Determine the number of different groups that can be selected which contain at least 3 beginners.

(2 marks)

Solution
Choose 3 beginners: $\binom{5}{3} \binom{7}{1} = 70$ Choose 4 beginners: $\binom{5}{4} \binom{7}{0} = 5$ Total: $70 + 5 = 75$
Specific behaviours
<ul style="list-style-type: none"> ✓ number with 3 beginners ✓ number with 4 beginners and total

(c) Determine the probability that the group contains

(i) no advanced members.

(2 marks)

Solution
$\binom{8}{4} \binom{4}{0} = 70 \rightarrow P = \frac{70}{495} = \frac{14}{99} = 0.\overline{14}$
Specific behaviours
<ul style="list-style-type: none"> ✓ calculates number ✓ states probability (no need to simplify)

(ii) exactly one intermediate member.

(2 marks)

Solution
$\binom{3}{1} \binom{9}{3} = 252 \rightarrow P = \frac{252}{495} = 0.509$
Specific behaviours
<ul style="list-style-type: none"> ✓ calculates number ✓ states probability (no need to simplify)

Question 20

(7 marks)

The equation $f(x) = k$ has just one solution, where $f(x) = ax^3 + bx^2 - 5x - 2$, and a, b and k are constants.

The graph of $y = f(x)$ cuts the x -axis at $x = 1$, $x = -2$, and at one other point.

Determine the value(s) of the constant k , rounded to 2 decimal places. Explain your reasoning.

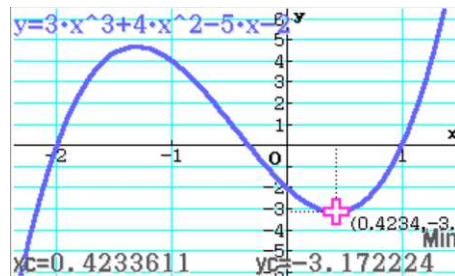
Solution

Use roots to solve for a and b :

$$f(1) = 0 \Rightarrow a + b - 5 - 2 = 0$$

$$f(-2) = 0 \Rightarrow -8a + 4b + 10 - 2 = 0$$

Solving simultaneously with CAS gives $a = 3$ and $b = 4$.



For one solution, k must be greater than the local maximum or less than the local minimum of $f(x)$ - found using CAS.

Local maximum is $y = 4.6702$

Local minimum is $y = -3.1722$

Hence $k < -3.17$ or $k > 4.67$.

Specific behaviours

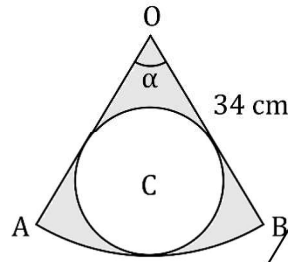
- ✓ indicates solving for a and b
- ✓ equates $f(1) = 0, f(-2) = 0$
- ✓ identifies equations as simultaneous
- ✓ solves equations for a and b
- ✓ describes case for one solution
- ✓ states value of local minimum, maximum
- ✓ correct inequalities for k

Question 21

(7 marks)

The diagram shows sector OAB of a circle centre O of radius 34 cm and $\alpha = 50^\circ$.

Circle C is inside the sector and just touches OA , OB and arc AB .



(a) Determine the area of sector OAB .

(3 marks)

Solution
$A = \frac{50^\circ}{360^\circ} \times \pi(34)^2 = \frac{1445\pi}{9} \approx 504 \text{ cm}^2$
Specific behaviours
<ul style="list-style-type: none"> ✓ indicates suitable method ✓ calculates area

(b) Show that the radius of circle C is 10.1 cm, correct to 1 decimal place.

(3 marks)

Solution
Extend sector sides to form isosceles triangle $OA'B'$.
$MB' = 34 \sin 25^\circ \approx 14.5 \text{ cm}$
$r = 14.5 - r \Rightarrow 2r = 14.5 \Rightarrow r = 7.25 \text{ cm}$
Specific behaviours
<ul style="list-style-type: none"> ✓ forms isosceles triangle ✓ calculates length of MB' ✓ calculates radius

Not Assessed

(c) Determine the area of the shaded region, inside sector OAB but outside circle C .

(2 marks)

Solution
$A_c = \pi(10.1)^2 \approx 320$
Shaded area = $504 - 320 = 184 \text{ cm}^2$
Specific behaviours
<ul style="list-style-type: none"> calculates area of circle calculates shaded area, with units

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

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Supplementary page

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